A Note on the Weighted Average Cost of Capital WACC

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Abstract

Most finance textbooks (See Benninga and Sarig, 1997, Brealey, Myers and Marcus, 1996, Copeland, Koller and Murrin, 1994, Damodaran, 1996, Gallagher and Andrew, 2000, Van Horne, 1998, Weston and Copeland, 1992) present the Weighted Average Cost of Capital WACC calculation as:

$$WACC = d(1-T)D\% + eE\% \qquad (1)$$

Where d is the cost of debt before taxes, T is the tax rate, D% is the percentage of debt on total value, e is the cost of equity and E% is the percentage of equity on total value. All of them precise (but not with enough emphasis) that the values to calculate D% y E% are market values. Although they devote special space and thought to calculate d and e, little effort is made to the correct calculation of market values. This means that there are several points that are not sufficiently dealt with: Market values, location in time, occurrence of tax payments, WACC changes in time and the circularity in calculating WACC. The purpose of this note is to clear up these ideas and emphasize in some ideas that usually are looked over.

Also, some suggestions are presented on how to calculate, or estimate, the equity cost of capital.

Keywords

Weighted Average Cost of Capital, WACC, firm valuation, capital budgeting, equity cost of capital.

JEL codes

D61: Cost-Benefit Analysis, G31: Capital Budgeting, H43: Project appraisal

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Introduction

Most finance textbooks (See Benninga and Sarig, 1997, Brealey, Myers and Marcus, 1996, Copeland, Koller and Murrin, 1994, Damodaran, 1996, Gallagher and Andrew, 2000, Van Horne, 1998, Weston and Copeland, 1992) present the Weighted Average Cost of Capital WACC calculation as:

$$WACC = d(1-T)D\% + eE\% \qquad (1)$$[4]

Where d is the cost of debt before taxes, T is the tax rate, D% is the percentage of debt on total value, e is the cost of equity and E% is the percentage of equity on total value. All of them precise (but not with enough emphasis) that the values to calculate D% y E% are market values. Although they devote special space and thought to calculate d and e, little effort is made to the correct calculation of market values. This means that there are several points that are not sufficiently dealt with:

1. Market values are calculated period by period and they are the present value at WACC of the future cash flows.
2. These values to calculate D% and E% are located at the beginning of period t, where the WACC belongs. From here on, the right notation will be used.
3. d(1-T) implies that the tax payments coincides in time with the interest payments. (Some firms could present this payment behavior, but it is not the rule. Only those that are subject to tax withheld from their customers, pay taxes as soon as they pay interest charges).

[1] This is part of a paper on firm valuation, forthcoming.
[2] Most part of this paper was written while Mr. Velez-Pareja was teaching at Universidad Javeriana, Bogotá, Colombia.
[3] Currently, Professor Tham is teaching at the Fulbright Economics Teaching Program (FETP) in Ho Chi Minh City, Vietnam.
[4] This formula is derived in Appendix B.

4. Because of 1., 2. and the existence of changing macroeconomic environment, (say, inflation rates) WACC changes from period to period.
5. That there exists circularity when calculating WACC. In order to know the firm value it is necessary to know the WACC, but to calculate WACC, the firm value and the financing profile are needed.
6. That we obtain full advantage of the tax savings in the same year as taxes and interest charges are paid. This means that earnings before interest and taxes (EBIT) are greater than or equal to the interest charges.
7. There are no losses carried forward.
8. That (1) implies a definition for e, the cost of equity, in this case,

$$e_t = \rho_t + (\rho_t - d)(1-T)D\%_{t-1}/E\%_{t-1} \qquad (2)$$[5]

In this expression, e_t is the levered cost of equity, ρ_t is the cost of unlevered equity, d is the cost of debt, T is the tax rate, $D\%_{t-1}$ is the proportion of debt on the total market value for the firm, at t-1 and $E\%_{t-1}$ is the proportion of equity on the total market value for the firm, at t-1. It can be shown that line 2 results from the assumption regarding the discount rate for the tax savings. In this case that rate is d and expression 2 is valid only for perpetuities. When working with n finite it can be shown that the expression for e changes for every period (see Velez-Pareja and Tham 2001c). The assumption behind d as the discount rate is that the tax savings are a non-risky cash flow.

The purpose of this note is to clear up these ideas and emphasize in some ideas that usually are looked over.

The Modigliani-Miller Proposal

The basic idea is that the firm value does not depend on how the stakeholders finance it. This is the stockholders (equity) and creditors (liabilities to banks, bondholders, etc.) The reader should examine this idea in an intuitive manner and she will find it reasonable. Because of this idea, Franco Modigliani and Merton Miller (MM from here on) were awarded the Nobel Prize in Economics. They proposed that with perfect market conditions, (perfect and complete information, no taxes, etc.) the capital structure does not affect the value of the firm because the equity holder can borrow and lend and thus determine the optimal amount of leverage. The capital structure of the firm is the combination of debt and equity in it.

[5] This formula is derived in Appendix C.

That is, V^L the value of the levered firm is equal to V^{UL} the value of the unlevered firm.

$$V^L = V^{UL} \qquad (3)$$

And in turn, the value of the levered firm is equal to_V^{Equity} the value of the equity plus V^{Debt} the value of the debt.

$$V^L = V^{Equity} + V^{Debt} \qquad (4)$$

What does it imply regarding the Weighted Average Cost of Capital WACC? Simple. If the firm has a given cash flow, the present value of it at WACC (the firm total value) does not change if the capital structure changes. If this is true, it implies that the WACC will remain constant no matter how the capital structure changes. This situation happens when no taxes exist. To maintain the equality of the unlevered and levered firms, the return to the equity holder (levered) must change with the amount of leverage (assuming that the cost of debt is constant)

One of the major market imperfections are taxes. When corporate taxes exist (and no personal taxes), the situation posited by MM is different. They proposed that when taxes exist the total value of the firm does change. This occurs because no matter how well managed is the firm, if it pays taxes, there exists what economists call an externality. When the firm deducts any expense, the government pays a subsidy for the expense. It is reflected in less tax. In particular, this is true for interest payments. The value of the subsidy (the tax saving) is TdD, where the variables have been defined above.
Hence the value of the firm is increased by the present value of the tax savings or tax shield.

$$V^L = V^{UL} + V^{TS} \qquad (5)$$

When a firm has debt there exists some other contingent or hidden costs associated to the fact to the possibility that the firm goes to bankruptcy. Then, there are some expected costs that could reduce the value of the firm. The existence of these costs deters the firm to take leverage up to 100%. One of the key issues is the appropriate discount rate for the tax shield. In this note, we assert that the correct discount rate for the tax shield is ρ, the return to unlevered equity, and the choice of ρ is appropriate whether the percentage of debt is constant or varying over the life of the project.

In this note the effects of taxes on the WACC will be studied. When calculating WACC two situations can be found: with or without taxes. In the first case, as said above, the WACC is constant, no matter how the firm value be split between creditors and stockholders. (The assumption is that inflation is kept constant, otherwise, the WACC should change accordingly). When inflation is not constant, WACC changes, but due to the inflationary component and not due to the capital structure. In this situation, WACC is the cost of the assets or the cost of the firm, ρ and at the same time is the cost of equity when unlevered. This means,

$$\rho_t = dD_{t-1}\% + eE_{t-1}\% \qquad (6)$$

This ρ is defined as the return to unlevered equity. The WACC is defined as the weighted average cost of debt and the cost of levered equity. In a MM world ρ is equal to WACC without taxes. When taxes exist, the WACC calculation will change taking into account the tax savings.
If it is true that the cost ρ, is constant, e, the cost of equity changes according to the leverage. Here for simplicity we assume that the ρ is constant, but this assumption is not necessary. If the ρ is changing then in each period, the WACC will change as well, not only for the eventual change in the financing profile, but for the change in ρ. In any case, e has to change in order to keep ρ constant or in order to be consistent with the changing ρ.
The cost of equity, e is

$$e_t = (\rho_t - d\ D\%_{t-1})/\ E\%_{t-1} = \rho_t + (\rho_t - d)D\%_{t-1}/E\%_{t-1} \quad (7)^6$$

This equation is proposed by Harris and Pringle (1985) and is part of their definition of WACC[7]. (Authors are working a paper where the expression for e is derived under different assumptions for the discount rate for the tax savings and for perpetuities and finite periods). Note the absence of the (1-T) factor.
As before, it can be shown that line 7 results from the assumption regarding the discount rate for the tax savings. In this case that rate is ρ and it can be shown that e, defined in line 7, is the same for any period length and for each period, including perpetuities[8]. The assumption behind ρ as the discount rate is that the tax savings are a strictly correlated to the free cash flow.

What is the meaning of line 7? Since ρ and d are constant, we see that the return to levered equity e is a linear function of the debt-equity ratio.

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[6] This formula is derived in Appendix C.
[7] This was the original proposal by M&M in a seminal paper published in 1958, but corrected in 1963.
[8] Vélez-Pareja and Tham, The Correct Derivation for the Cost of Equity in a MM World, April, 2001. Draft in process.

It should be no surprise that there is a positive relationship between e, the return to levered equity and the debt-equity ratio. Since the debt holder has a prior claim on the expected cash flow generated by the firm, relative to the debt holder, the risk to the equity holder is higher and the equity holder demands a higher return to compensate for the higher risk. The higher the amount of debt, the higher is the risk to the equity holder, who is the residual claimant.

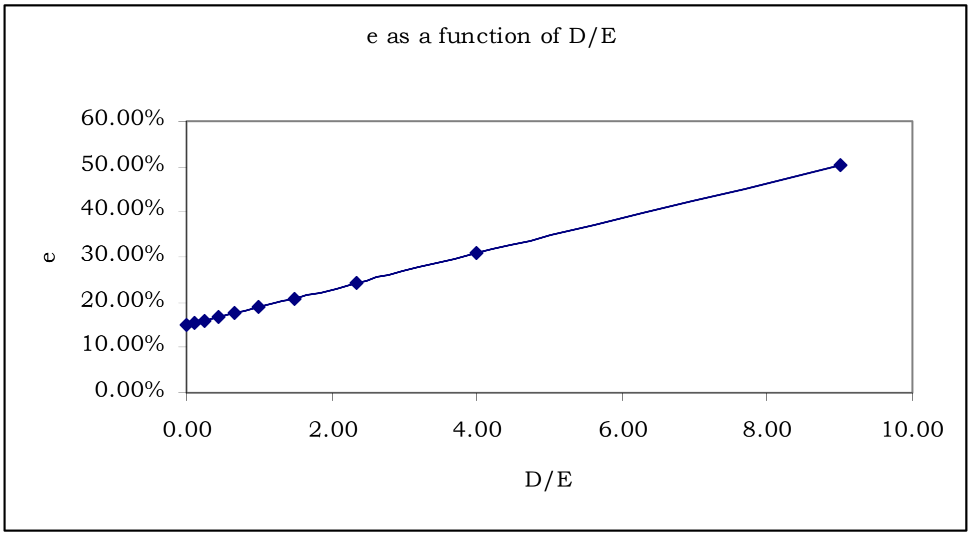
Line 7 shows the relationship between the e, the return to levered equity and the debt-equity ratio. The following table shows the relationship between D, the amount of debt, the debt-equity ratio, E, the amount of equity and e, the return to levered equity.

Table 1: Relationship between D, the amount of debt, the debt-equity ratio and e, the return to levered equity for ρ = 15.1% and d=11.2%

Debt, D	Equity, E	D/E Ratio	E
0	1000	0.00	15.10%
100	900	0.11	15.53%
200	800	0.25	16.08%
300	700	0.43	16.77%
400	600	0.67	17.70%
500	500	1.00	19.00%
600	400	1.50	20.95%
700	300	2.33	24.20%
800	200	4.00	30.70%
900	100	9.00	50.20%

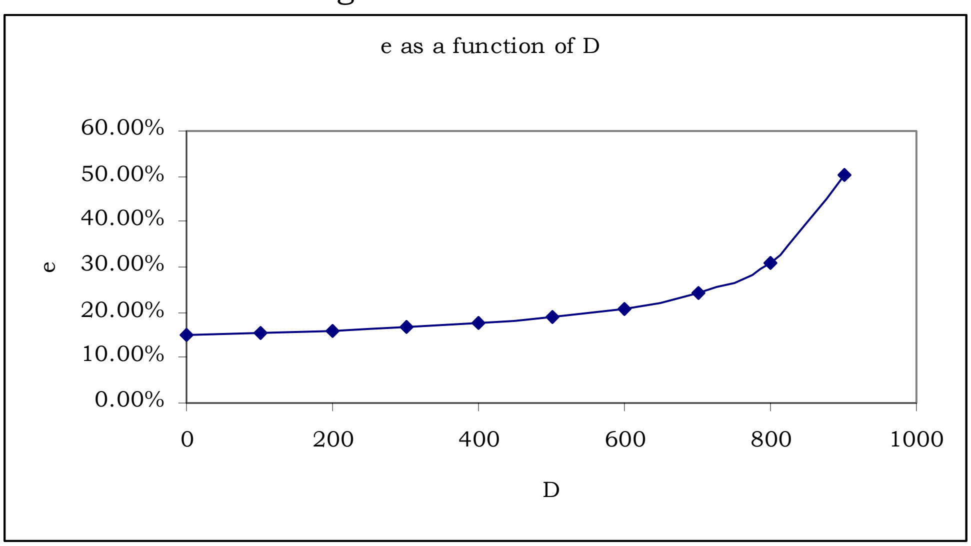
If the amount of debt is $100, the debt-equity ratio is 0.11 and the return to levered equity is 15.53%%. Note that there is a linear relationship between e, the return to levered equity and the debt-equity ratio.

Figure 1. e as a function of D/E



If the amount of debt increases from 100 to 200, the return to levered equity increases by 0.43 percentage points, from 15.1% to 15.53%. However, the relationship between e, the return to levered equity and the amount of debt D is non-linear (remember that E = Total value – D). If the amount of debt increases from 500 to 600, the return to levered equity increases by 1.95 percentage points, from 19% to 20.95%.

Figure 2. e as a function of D



Hence, WACC after taxes will be calculated as

$$WACC_t = d_t(1\text{-}T)\ D\%_{t-1} + e_t E\%_{t-1} = \rho - dTD\% \qquad (8)$$

The values for D% y E% have to be calculated on the total value of the firm for the beginning of each period. This is the well known expression for the weighted average cost of capital.
Based on (8), a new presentation of WACC is proposed:

$$\text{Adjusted WACC} = \rho - TS/TV \qquad (9)$$

Where TS means tax savings and TV is the total levered value of the firm. This means that dTD% is the same as dTD/TV and in general, we call TS to the tax savings -dDT. However, it must be said that the tax savings are equal to dDT only when taxes and interest charges are paid in the same year as accrued. The implicit assumption in (9) is that we consider the actual tax savings earned and when they occur. This new version of WACC has the property to give the same results as (8) and what is most important, as TS is the real tax savings earned, it takes into account the losses carried forward (LCF), when they occur. This problem has been studied in two papers that are being processed by the authors. See Velez-Pareja and Tham (2001a and 2001b).
If the Capital Asset Pricing Model (CAPM) is used, it can be demonstrated that there is a relationship between the betas of the components (debt and equity) in such a way that

$$\beta_{t\ firm} = \beta_{t\ debt}\ D_{t-1}\% + \beta_{t\ stock}\ E_{t-1}\% \qquad (10)$$

If $\beta_{t\ stock}$, $\beta_{t\ debt}$, $D_{t-1}\%$ and $E_{t-1}\%$ are known, then ρ can be calculated as

$$\rho = R_f + \beta_{t\ firm}\ (R_m - R_f) \qquad (11)$$

Where R_f is the risk free rate of return and R_m is the market return and $(R_m - R_f)$ is the market risk premium. And this means the ρ can be calculated for any period.

Calculations for e and ρ

The secret is to calculate e or ρ. If e is known for a given period, the initial period, for instance, ρ can be calculated. On the contrary, if ρ is known e can be calculated. For this reason several options to calculate e and ρ are presented.
In order to calculate e, we have several alternatives:

1. With the Capital Asset Pricing Model, CAPM. This is the case of a firm that is traded at the stock exchange, it is traded on a regularly

basis and we think the CAPM works well. However, it has to be said that if we know the value of the equity (it is traded at the stock exchange) it is not necessary to discount the cash flows to calculate the value.

2. With the Capital Asset Pricing Model, CAPM adjusting the betas. This is the case for a firm that is not registered at the stock exchange or if registered, is not frequently traded and we believe the model works well. It is necessary to pick a stock similar to the one we are studying, (from the same industrial sector, about the same size and about the same leverage). This is called the proxy firm.

Example:

The beta adjustment is made with[9]

$$\beta t = \beta_{proxy} \frac{\left[1 + \frac{D_{nt}}{E_{nt}}(1-T)\right]}{\left[1 + \frac{D_{proxy}}{E_{proxy}}(1-T)\right]} \qquad (12)$$

Where, β_{nt} is the beta for the stock not registered at the stock exchange; D_{nt} is the market value of debt, E_{anb} is the equity for the stock not registered in the stock; D_{proxy} is the market value of debt for the proxy firm, E_{proxy} is the market value of equity for the proxy firm.

For instance, if you have a stock traded at the stock exchange and the beta is β_{proxy} of 1.3, a debt D_{proxy} of 80, E_{proxy} worth 100, and we desire to estimate the beta for a stock not registered in the stock exchange. This non-traded stock has a debt D_{nt} of 70 and equity of E_{nt} of 145 and a tax rate of 35%, and then beta for the non-traded stock can be adjusted as

$$\beta_{nt} = \beta_{proxy} \frac{\left[1 + \frac{D_{nt}}{E_{nt}}(1-T)\right]}{\left[1 + \frac{D_{proxy}}{P_{proxy}}(1-T)\right]} = 1.3 \frac{\left[1 + \frac{70}{145}(1-35\%)\right]}{\left[1 + \frac{80}{100}(1-35\%)\right]} = 1.12$$

This is easier said than done. If we have illustrated the use of the formula, we have to recall that the equity for the non traded firm is not known. That value is what we are looking for. Hence, there will be a circularity when using this approach.

[9] Based on Robert S. Hamada, "Portfolio Analysis, Market Equilibrium and Corporation Finance", Journal of Finance, 24, (March, 1969), pp. 19-30.

3. Subjectively and assisted by a methodology such as the one presented by Cotner and Fletcher, 2000 and applied to the owner of the firm. With this approach the owner given a leverage level estimates the perceived risk. This risk premium is added to the risk free rate and the result would be an estimate for e.
4. Subjectively as 3., but direct. This is, asking the owner, for a given value level of debt and a given cost of debt, what is the required return to equity?
5. An estimate based on book value (given that these values are adjusted either by inflation adjustments or asset revaluation, so the book value is a good proxy to the market value).

An example: Assume a privately held firm. Tax rate is 35%

Table 2. Financial information of hypothetical firm

Year	Adjusted book value for equity E	Dividends paid D	Return R_t $((D_t + E_t)/E_{t-1} - 1$
1990	$1,159	$63	
1991	$1,341	$72	21.92%
1992	$2,095	$79	62.12%
1993	$1,979	$91	-1.19%
1994	$3,481	$104	81.15%
1995	$4,046	$126	19.85%
1996	$3,456	$176	-10.23%
1997	$3,732	$201	13.80%
1998	$4,712	$232	32.48%
1999	$4,144	$264	-6.45%
2000	$5,950	$270	50.10%

Table 3 Additional macroeconomic information

Year	Nominal risk Free rate of interest[10] R_f	CPI	Inflation i_f $(CPI_t/CPI_{t-1})-1$	Real interest rate $i_r = (1+R_f)/(1+i_f)-1$	Return to equity $e_t= ((D_t+E_t)/E_{t-1})-1$	Risk premium $i_\theta = e_t - R_{ft} \times (1-T)$
1990	36.3%	166.94				
1991	30.6%	211.72	26.8%	3.0%	21.92%	2.0%
1992	28.9%	264.94	25.1%	3.0%	62.12%	43.3%
1993	26.3%	324.84	22.6%	3.0%	-1.19%	-18.3%
1994	26.3%	398.24	22.6%	3.0%	81.15%	64.1%
1995	15.8%	475.76	19.5%	-3.1%	19.85%	9.6%
1996	16.3%	578.71	21.6%	-4.4%	-10.23%	-20.8%
1997	21.2%	681.06	17.7%	3.0%	13.80%	0.0%
1998	51.7%	794.80	16.7%	30.0%	32.48%	-1.1%
1999	16.4%	898.12	13.0%	3.0%	-6.45%	-17.1%
2000	12.9%	984,34	9.6%	3.0%	50.10%	41.7%
2001			Expected 10%	Average 4.4%		Average 10.3%

Estimated risk free rate for 2001:

$R_{f\,2001} = ((1+i_{f\,est.})(1+i_{r\,avg.}) - 1) \times (1-T) = ((1+10\%)(1+4.4\%) - 1) \times (1-0.35) = 9{,}61\%$

Cost of equity e = $R_{f\,2001} + i_{\theta\,average} = 9{,}61\% + 10{,}30\% = 20{,}0\%$

6. Calculate the market risk premium as the average of $R_m - R_f$, where R_m is the return of the market based upon the stock exchange index and R_f is the risk free rate (say, the return of treasury bills or similar). Then, subjectively, the owner could estimate if he prefers, in terms of risk, to stay in the actual business or to buy the stock exchange index basket. If the actual business is preferred, then one could say that the beta of the actual business is lower than 1, the market beta, and the risk perceived is lower than the market risk premium, $R_m - R_f$. This is an upper limit for the risk premium of the owner. This upper limit could be compared with zero risk premium, the risk free rate risk premium which is the lower limit for the risk perceived by the equity owner.

If the owner prefers to buy the stock exchange index basket, we could say that the actual business is riskier than the market. Then, the beta should be greater than 1 and the perceived risk for the actual business should be greater that $R_m - R_f$.

In the first case, the owner could be confronted with different combinations -from 0% to 100%- of the stock exchange index basket and the risk free investment and the actual business. After several trials, the

[10] This information is based on real data for real nominal risk free rates in the Colombian bond market.

owner eventually will find the indifference combination of risk free and the stock exchange index basket. The perceived risk could be calculated as a weighted risk, or simply, the market risk premium (R_m - R_f) times the proportion of the stock exchange index basket accepted. In fact what has been found is the beta for the actual business.

In the second case one must choose the highest beta found in the stock exchange index basket. (The stock exchange or any governmental control office usually calculates these betas. In Colombia the betas for each stock are calculated by the *Superintendencia de Valores*, similar to the US Securities Exchange Commission, SEC). This beta should be used to multiply the market risk premium R_m - R_f, and the result would be an estimate of the risk premium for the riskiest stock in the index. This might be an upper limit for the risk perceived by the owner. In case this risk is lower that the perceived risk by the owner, it might be considered as the lower limit. In case that the riskier stock is considered riskier than the actual business, then the lower limit is the market risk premium, R_m - R_f. In this second case, the owner could be confronted with different combinations -from 0% to 100%- of the stock exchange index basket and the riskiest stock and the actual business. After several trials, the owner eventually will find the indifference combination of risk free and the stock exchange index basket. The perceived risk could be calculated as a weighted risk. That is, the market risk premium (R_m - R_f) times the proportion of the stock exchange index basket accepted plus the risk premium for the riskiest stock in the index (its beta times the market risk premium, R_m - R_f) times the proportion accepted for that stock.

In both cases the result might be an estimation of the risk premium for the actual business. This risk premium could be added to the risk free rate (using Fisher Theorem), and this might be a rough estimate of e.

If e, D% and E% are known, then ρ is calculated with (6). As it is necessary to know the market values that are the result of discounting the future cash flows at WACC, then circularity is found, but it is possible to solve it with a spreadsheet.

Another option is to calculate ρ directly. One of the following alternatives could be used:

1. Using the CAPM and unlevering the beta and using line (13), which is derived from (12).

$$\beta t = \frac{\beta_{proxy}}{\left[1+\frac{D_{proxy}}{E_{proxy}}(1-T)\right]} \qquad (13)$$

With this beta we apply CAPM to obtain ρ.

2. According to MM, the WACC before taxes (ρ) is constant and independent from the capital structure of the firm. Then we could ask the owner for an estimate on how much she is willing to earn assuming no debt. A hint for this value of e could be found looking how much she could earn in a risk free security when bought in the "secondary" market. On top of this, a risk premium, subjectively calculated must be included.
3. Another way to estimate ρ is assessing subjectively *the risk for the firm* and this risk could be used to calculate ρ using the Fisher Theorem with the risk free rate. (Cotner and Fletcher, 2000 present a methodology to calculate the risk of a firm not publicly held[11]). This methodology might be applied to the managers and other executives of the firm. This would give the risk premium for the firm. As this risk component would be added to the risk free rate, the result is ρ calculated in a subjective manner. A hint that could help in the process is to establish minimum or maximum levels for this ρ. The minimum could be the cost of debt before taxes. The maximum could be the cost of opportunity of the owners, if it is perceptible (that is, if it has been "told" by them or if, by observation, it is known observing were they are investing (other investments made by them).

This ρ is in accordance to the actual level of debt. It has to be remembered that ρ is, according to MM, constant and independent from the capital structure of the firm.

This ρ is named in other texts as K_A cost of the assets or the firm, (for instance, Ruback, 2000) or K_u cost of unlevered equity (for instance, Fernandez, 1999a y 1999b).

If ρ is estimated directly and we wish to estimate the WACC (or the e), then circularities will be present. However, as will be shown below, the total value of the firm can be calculated with ρ, using the Capital Cash Flow, CCF, and no circularities will be present and here is no need to calculate the leverage ratio for every period.

An Example for Calculating WACC and the Firm Value

For a better understanding of these ideas, an example is presented. This example is done using the Harris Pringle formulation. In this example it is assumed that ρ is the correct discount rate for tax savings. For

[11] In fact, in the article the authors say that the methodology is to calculate the risk of the cost of capital, although at the end they say it is to define the risk for the equity cost. The way the methodology is presented allows thinking that it is the firm risk that is dealt with and this risk is added to the risk free rate. With this, the cost of capital before taxes for the firm is found. This would be ρ

illustration purposes, in an annex the other assumption is used, this is, d as the discount rate.

Assume a firm with the following information:

The cost of the unlevered equity ρ	15.1%
Debt's beta	0.2
Risk free rate	10%
Market risk premium	6%
Tax rate	35%

Using CAPM the cost of debt is 10% + 0.2x6% = 11.2% before taxes. If the debt is not traded, the cost is the one stipulated in the contract.

The information about the investment, free cash flows, debt balances and initial equity is

Table 4 Free cash flow and initial investment

Year	0	1	2	3	4
Free cash flow FCF[12]		170,625.00	195,750.00	220,875.00	253,399.45
Debt at end of period, D	375,000.00	243,750.00	75,000.00	37,500.00	
Initial equity investment	125,000.00				
Total initial investment	500,000.00				

The WACC calculations are made estimating the debt and equity participation in the total value of the firm for each period and calculating the contribution of each to the WACC after taxes. As a first step, we will not add up these components to find the value of WACC and we will calculate the total firm value with the WACC set at 0. We will construct each table, step by step, assuming that WACC is zero. Remember that $D_{t-1}\% = D/TV_{t-1}$

[12] In the FCF at year 4 we assume there is a terminal value, that takes into account the value added by the firm from year 5 to infinity. This is a very important issue in firm valuation because experience shows that more than 50% of the firm value is provided by terminal value. The subject is not addressed in detail because it is beyond the scope of this paper. It is a complex issue and the purpose of this text is to illustrate how to involve market values in the calculation of WACC. The interested reader can read a note on this topic at www.poligran.edu.co/decisiones., Vélez-Pareja and Tham, 2001.

Table 5 WACC calculation. Contribution of debt to WACC.

Year	0	1	2	3	4
Debt					
Relative weight of debt D% (Debt balance)/Total value of firm at t-1)	44.61%	36.38%	15.81%	14.80%	
Cost of debt after taxes d(1-T)		7.28%	7.28%	7.28%	7.28%
Contribution of debt to WACC d(1-T)D%		3.25%	2.65%	1.15%	1.08%

The same procedure is used to estimate the contribution of equity to WACC.

Table 6 WACC calculation. Contribution of equity to WACC.

Year	0	1	2	3	4
Equity					
Relative weight of equity E% = (1-D%)	55.39%	63.62%	84.19%	85.20%	
Cost of equity e = (ρ_t - d $D\%_{t-1}$)/ $E\%_{t-1}$		18.24%	17.33%	15.83%	15.78%
Contribution of equity to WACC = E%xe		10.10%	11.03%	13.33%	13.44%

Done this, our table for WACC and Total Value will appear as

Table 7 WACC calculations

Year	0	1	2	3	4
WACC after taxes (Debt + equity contributions)					
Total value TV, at t-1 and WACC = 0	840,649.45	670,024.45	474,274.45	253,399.45	

Example: Firm value at end of year 3 is

253,399.45/(1+$WACC_4$)= 253,399.45/(1+0%) = 253,399.45.

For year 2 it will be

(253,399.45 + 220,875.00)/(1+$WACC_3$) = (253,399.45 + 220,875.00) /(1+0%) = 474,274.45

and so on for the other years.

It is recommended that the last arithmetic operation be WACC calculation as the sum of the debt and equity contribution to the cost of capital.
At this point we recommend to set the spreadsheet to handle circularities following these instructions:

1. Select the option *Tools* in the textual menu in Excel.
2. Select *Options*
3. Select the tab *Calculate*.
4. In the dialog box select *Iteration* and click *Ok*.

This procedure can be done before starting the work in the spreadsheet or when Excel declares the presence of circularity. After these instructions are done, then, the WACC can be calculated as the sum of the debt and equity contribution to the cost of capital.
Now we can proceed to formulate the WACC as the sum of the two components: debt contribution and equity contribution. When the WACC is calculated, then Table 5 will be shown as

Table 8 WACC calculation. Contribution of debt to WACC (final).

Year	0	1	2	3	4
Debt					
Relative weight of debt D% (Debt balance)/Total value of firm at t-1)	61.68%	47.38%	19.39%	16.94%	
Cost of debt after taxes d(1-T)		7.28%	7.28%	7.28%	7.28%
Contribution of debt to WACC d(1-T)D%		4.49%	3.45%	1.41%	1.23%

The same procedure is used to estimate the contribution of equity to WACC.

Table 9 WACC calculation. Contribution of equity to WACC (final).

Year	0	1	2	3	4
Equity					
Relative weight of equity E% = (1-D%)	38.32%	52.62%	80.61%	83.06%	
Cost of equity $e = (\rho_t - d\ D\%_{t-1}) / E\%_{t-1}$		21.38%	18.61%	16.04%	15.90%
Contribution of equity to WACC = E%xe		8.19%	9.79%	12.93%	13.20%

Note that the cost of equity –e– is larger than ρ as expected, because ρ is the cost of the stockholder, as if the firm were unlevered[13]. When there is debt –e calculation– necessarily e ends up being greater than ρ, because of leverage. With these values it is possible to calculate the firm value for each period.
If e_1 is known, as it was said above, ρ is found with (6). Excel solves the circularity that is found and the same values result.

Table 10 WACC calculations (final)

Year	0	1	2	3	4
WACC after taxes (Debt + equity contributions)		12.7%	13.2%	14.3%	14.4%
Firm value a end of t	607,978.04	514,457.73	386,835.85	221,433.06	

Notice that WACC results in a lower value than ρ. WACC is after taxes.

Example: Firm value at end of year 3 is

$253{,}399.45/(1+WACC_4) = 253{,}399.45/(1+14.4\%) = 221{,}433.06$.

For year 2 it will be

$(221{,}433.06 + 220{,}875.00)/(1+WACC_3) = (221{,}433.06 + 220{,}875.00)/(1+14.3\%) = 386{,}835.85$

and so on for the other years.

The reader has to realize that the values 14.4% and 14.3%, etc. are not calculated from the beginning because they depend on the firm value that is going to be calculated with the WACC. In this case circularity is generated. This is solved allowing the spreadsheet to make enough iteration until it finds the final numbers.
With the WACC values for each period the present value of future cash flows and the NPV are calculated.

[13] As MM say that ρ is constant and independent from the capital structure, it will be equal to ρ when debt is zero. This ρis WACC before taxes. And this is the condition for the validity of the first proposition of MM.

Table 11 NPV calculations

Year	0	1	2	3	4
Present value of cash flows	607,978.04	151,421.50	153,403.90	151,385.08	151,767.56
NPV	107,978.04				

For instance, the present value for the cash flow at year 4 is

253,399.45/((1+$WACC_4$)(1+$WACC_3$)(1+$WACC_2$)(1+$WACC_1$))
253,399.45/((1+14.4%)(1+14.3%)(1+13.2%)(1+12.7%)) = 151,767.56

Now all the present values are added and the total present value is obtained. If the initial investment is 500,000, then, NPV is 107,978.04.
Using the MM approach on the case with taxes the same result can be reached calculating the present value for the free cash flow assuming no debt and discount it a ρ, or what is the same, at WACC before taxes and add up the present value of tax savings at the same rate of discount, ρ. Myers proposed this in 1974 and it is known as Adjusted Present Value APV. Myers and all the finance textbooks teach that the discount rate should be the cost of debt. However, the tax savings depend on the firm profits. Hence, the risk associated to the tax savings is the same as the risk of the cash flows of the firm rather than the value of the debt. Hence, the discount rate should be ρ. For this reason the tax savings are also discounted at ρ. This way, the present value for the free cash flows discounted at WACC after taxes coincides with the present value of the free cash flow assuming no debt discounted at ρ and added with the present value of the tax savings discounted at the same ρ.
The use of ρ to discount the tax savings has been proposed by Tham, 1999, Tham, 2000 and Ruback, 2000. Tham proposes to add to the total value of the firm (the present value of the FCF at ρ), the present value of the tax savings discounted at ρ. Ruback presents the Capital Cash Flow and discount it at ρ. The CCF is simply the FCF plus the tax savings so,

$$CCF = FCF + Tax\ savings \quad (11)$$

$$\begin{aligned} &PV(FCF\ at\ WACC\ after\ taxes) \\ &= PV(FCF\ without\ debt\ at\ \rho) + PV(Tax\ savings\ at\ \rho) \quad (12) \\ &= PV(CCF\ at\ \rho) \end{aligned}$$

Table 12 Calculation of value and APV with ρ

Year	0	1	2	3	4
Interest payments		42,000.00	27,300.00	8,400.00	4,200.00
Tax savings TS = TxI		14,700.00	9,555.00	2,940.00	1,470.00
Capital Cash Flow (CCF) = FCF + Tax savings		185,325.00	205,305.00	223,815.00	254,869.45
ρ		15.10%	15.10%	15.10%	15.10%
PV (CCF) at ρ		607,978.04			
Adjusted NPV (APV)					
PV(FCF at ρ)		585,228.51			
PV(TS at ρ)		22,749.53			
PV(FCF at ρ) + PV(TS at ρ)		607,978.04			
Adjusted NPV		107,978.04			

Notice that the same result is reached with the three methods.

From the point of view of firm valuation, the value is calculated with the present value of the free cash flow discounted at WACC minus the debt at 0. This value also can be reached with the equity cash flow (CFE) and it is equal to

$$\text{CFE} = \text{FCF} + \text{TS} - \text{Cash flow to debt before taxes CFD} \qquad (13)$$

Table 13. Calculating the value of equity with CFE

Year	0	1	2	3	4
CFE		12,075.00	9,255.00	177,915.00	213,169.45
PV(CFE at e)	232,978.04	9,948.31	6,428.52	106,499.41	110,101.80

When the present value of CFE at e, is calculated the same result is obtained. This is, 607,978.04 – 375,000 = 232,978.04. This means that the right discount rate to discount the CFE is e, and its discounted value is consistent with the value calculated with the FCF.

In table 13 we calculated the market value of equity using the market value calculated before. However, this is not an independent method when we use the values from other method. In order to calculate the market value of equity in an independent way we Hill use the same procedure utilized for the calculation with WACC. The difference is that we will calculate again the value of e. The first table with e equal to zero is

Table 14. Initial table to calculate the market equity value

Year	0	1	2	3	4
Free Cash Flow FCF		170,625.00	195,750.00	220,875.00	253,399.45
Interest charges		42,000.00	27,300.00	8,400.00	4,200.00
Debt payment		131,250.00	168,750.00	37,500.00	37,500.00
CFD		173,250.00	196,050.00	45,900.00	41,700.00
Tax savings TS		14,700.00	9,555.00	2,940.00	1,470.00
CFE =FCF-CFD+TS		12,075.00	9,255.00	177,915.00	213,169.45
Relative weigh to debt D%	47.6%	37.8%	16.1%	15.0%	
Relative weigh to equity E%	52.4%	62.2%	83.9%	85.0%	
E					
Debt at end of period	375,000.00	243,750.00	75,000.00	37,500.00	-
Market value of equity	412,414.45	400,339.45	391,084.45	213,169.45	
Total value	787,414.45	644,089.45	466,084.45	250,669.45	-

The final table for this calculation is as follows,

Table 15. Independent calculation of market equity value

Year	0	1	2	3	4
Free Cash Flow FCF		170,625.00	195,750.00	220,875.00	253,399.45
Interest charges		42,000.00	27,300.00	8,400.00	4,200.00
Debt payment		131,250.00	168,750.00	37,500.00	37,500.00
CFD		173,250.00	196,050.00	45,900.00	41,700.00
Tax savings TS		14,700.00	9,555.00	2,940.00	1,470.00
CFE=FCF-CFD+TS		12,075.00	9,255.00	177,915.00	213,169.45
Relative weight of debt D%	61.7%	47.4%	19.4%	16.9%	
Relative weight of equity E%	38.3%	52.6%	80.6%	83.1%	
e		21.4%	18.6%	16.0%	15.9%
Debt at end of period	375,000.00	243,750.00	75,000.00	37,500.00	-
Market value of equity	232,966.72	270,700.01	311,831.45	183,931.38	
Total value	607,966.72	514,450.01	386,831.45	221,431.38	-

Observe that working independently we reach the same values for equity, total value and e.

In summary, the different methodologies presented to calculate the total value of the firm are[14]:
Total Value for the firm TV = PV(FCF at WACC)
Total Value for the firm TV = PV(FCF at ρ) + PV(TS at ρ)
Total Value for the firm TV = PV(CCF at ρ).
Market value of equity E_{mv} = TV - D
Market value of equity E_{mv} = PV(CFE at e).
All these calculations should coincide.
In this example,

Table 14 A comparison of values by different approaches

Method	Total Value	Equity Value = Total Value - Debt
PV(FCF at $WACC_{t.}$)	607,978.04	232,978.04
PV(FCF at ρ) + PV(Tax savings at ρ)	607,978.04	232,978.04
PV(FCF+TS at ρ)	607,978.04	232,978.04
PV(CFE at e)		232,978.04

[14] There exist other methodologies, but they do not coincide among them. See Taggart, 1991

The value of equity is the price that the owners would sell their participation in the firm and this is higher than the initial equity contribution of 125,000.

Conclusions

The misuse of WACC might be due to several reasons. Traditionally there have been not computing tools to solve the circularity problem in WACC calculations. Now it is possible and easy with the existence of spreadsheets. Not having these computing resources in the previous years it was necessary to use simplifications such as calculating just one single discount rate or in the best of cases to use the book values in order to calculate the WACC.

Here a detailed (but known) methodology to calculate the WACC has been presented taken into account the market values in order to weigh the cost of debt and the cost of equity. By the same token a methodology based on the WACC before taxes ρ, constant (assuming stable macroeconomic variables, such as inflation) that does not depend on the capital structure of the firm has been presented.

The most difficult task is the estimation of ρ, or alternatively, the estimation of e. Here, a methodology to estimate those parameters is suggested. If it is possible to estimate ρ from the beginning, it will be possible to calculate the total and equity value independently from the capital structure of the firm, using the CCF approach or the Adjusted Present Value approach and discounting the tax savings at ρ.

In summary, the different methodologies presented to calculate the total value of the firm are:

Table 15 Summary

Method	Total Value	Equity Value
PV(FCF at $WACC_{t.}$)	607,978.04	232,978.04
PV(FCF at ρ) + PV(Tax savings at ρ)	607,978.04	232,978.04
PV(FCF+TS at ρ)	607,978.04	232,978.04
PV(CFE at e)		232,978.04

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Appendix A

An Example for Calculating WACC and the Firm Value

In this example it is assumed that d is the correct discount rate for tax savings.

Assume a firm with the following information:

The cost of the unlevered equity ρ	15.1%
Debt's beta	0.2
Risk free rate	10%
Market risk premium	6%
Tax rate	35%

Using CAPM the cost of debt is 10% + 0.2x6% = 11.2% before taxes. If the debt is not traded, the cost is the one stipulated in the contract.

The information about the investment, free cash flows, debt balances and initial equity is

Table A1 Free cash flow and initial investment

Year	0	1	2	3	4
Free cash flow FCF		170,625.00	195,750.00	220,875.00	253,399.45
Debt at end of period, D	375,000.00	243,750.00	75,000.00	37,500.00	
Initial equity investment	125,000.00				
Total initial investment	500,000.00				

The WACC calculations are made estimating the debt and equity participation in the total value of the firm for each period and calculating the contribution of each to the WACC after taxes.

Table A2 WACC calculation. Contribution of debt to WACC.

Year	0	1	2	3	4
Debt					
Relative weight of debt D% (Debt balance)/Total value of firm at t-1)	61.37%	47.04%	19.33%	16.91%	
Cost of debt after taxes d(1-T)		7.28%	7.28%	7.28%	7.28%
Contribution of debt to WACC d(1-T)D%		4.47%	3.42%	1.41%	1.23%

The same procedure is used to estimate the contribution of equity to WACC.

Table A3 WACC calculation. Contribution of equity to WACC.

Year	0	1	2	3	4
Equity					
Relative weight of equity E% = (1-D%)	38.63%	52.96%	80.67%	83.09%	
Cost of equity $e = \rho_t + (\rho_t - d)(1-T)D\%_{t-1}/E\%_{t-1}$		21.38%	17.40%	15.75%	15.65%
Contribution of equity to WACC = E%xe		8.26%	9.22%	12.70%	13.01%

Note that the cost of equity –e– is larger than ρ as expected, because ρ is the cost of the stockholder, as if the firm were unlevered[15]. When there is debt –e calculation– necessarily e ends up being greater than ρ, because of leverage. With these values it is possible to calculate the firm value for each period.
If e_1 is known, as it was said above, ρ is found with (6). Excel solves the circularity that is found and the same values result.

Table A4 WACC calculations

WACC calculations					
Year	0	1	2	3	4
WACC after taxes (Debt + equity contributions)		12.7%	12.6%	14.1%	14.2%
Firm value a end of t	611,056.56	518,200.45	387,957.75	221,818.63	

Notice that WACC results in a lower value than ρ. WACC is after taxes. Example: Firm value at end of year 3 is

$253,399.45/(1+WACC_4) = 253,399.45/(1+14.2\%) = 221,818.63$.

For year 2 it will be

$(221,818.63 + 220,875.00)/(1+WACC_3) = (221,818.63 + 220,875.00)/(1+14.1\%) = 387,957.75$

and so on for the other years.
The reader has to remember that the values 14,2% and 14,1%, etc. are not calculated from the beginning because they depend on the firm value that is going to be calculated with the WACC. In this case circularity is generated. This is solved allowing the spreadsheet to make enough iteration until it finds the final numbers. It is recommended that the last

15 As MM say that ρ is constant and independent from the capital structure, it will be equal to ρ when debt is zero. This ρis WACC before taxes. And this is the condition for the validity of the first proposition of MM.

arithmetic operation be WACC calculation as the sum of the debt and equity contribution to the cost of capital.
With the WACC values for each period the present value of future cash flows and the NPV are calculated.

Table A5 NPV calculations

Year	0	1	2	3	4
Present value of cash flows	611,056.56	151,361.31	154,161.64	152,441.17	153,092.43
NPV	111,056.56				

Table A6 Calculation of value and APV with d

Year	0	1	2	3	4
Interest payments		42,000.00	27,300.00	8,400.00	4,200.00
Tax savings TS = TxI		14,700.00	9,555.00	2,940.00	1,470.00
Adjusted NPV (APV)					
PV(FCF at d)		584,816.11			
PV(TS at d)		24,046.12			
PV(FCF at ρ) + PV(TS at d)		608,862.22			
Adjusted NPV		108,862.22			

Notice the different results with the two methods or assumptions.

From the point of view of firm valuation, its value is calculated with the present value of the free cash flow discounted at WACC minus the debt at 0. This value should be reached with the equity cash flow (CFE) and it is equal to

$$CFE = FCF + TS – \text{Cash flow to debt before taxes } CFD \qquad (13)$$

Table A7. Calculating the value of equity with CFE

Year	0	1	2	3	4
CFE		12,075.00	9,255.00	177,915.00	213,169.45
PV(CFE at e)	236,056.56	9,948.10	6,494.57	107,865.76	111,748.13

When the present value of CFE at e, is calculated the same result is obtained if calculated from the present value of the FCF at WACC. This is, 611,056.56 – 375,000 = 236,056.56. However, if the APV is used, the results differ.

APPENDIX B

Traditional WACC for a finite stream of free cash flow (FCF)

In this appendix, we derive the traditional WACC for a finite stream of free cash flow. Consider a finite stream of cash flows where FCF(i) is the free cash flow in year i. Similarly, CFE(i) is the cash flow to equity in year i, CFD(i) is the cash flow to debt in year i, and TS(i) is the tax shield in year i, based on the value of the debt at the end of the previous year i-1.

In any year i, the capital cash flow (CCF) is equal to the sum of the free cash flow and the tax shield.

$$CCF(i) = FCF(i) + TS(i) \qquad \text{(B1)}$$

Also, in any year i, the capital cash flow is equal to the sum of the cash flow to equity and the cash flow to debt.

$$CCF(i) = CFE(i) + CFD(i) \qquad \text{(B2)}$$

Combining line A1 and line A2, we obtain,

$$FCF(i) + TS(i) = CFE(i) + CFD(i) \qquad \text{(B3)}$$

Returns and taxes

The return to unlevered equity in year i is ρ_i, the return to unlevered equity in year i is e_i, the cost of debt in year i is d_i and the discount rate for the tax shield in year i is ψ_i. We assume only corporate tax τ. Furthermore, the corporate tax rate is constant. If the debt is risk-free, then the cost of debt is equal to the risk-free rate r_f.

Leverage and risk of tax shield

With respect to the leverage, we can make two assumptions. If the amount of debt is fixed, then the amount of the tax shield is known. In addition, if we assume that the tax shield is always realized, then the discount rate for the tax shield is equal to the risk-free rate r_f.

If the debt is a fixed percentage θ of the levered market value, then the amount of debt is unknown and consequently the amount of the tax shield is also unknown. If we assume that the tax shield is always realized, then the one-year ahead discount rate for the tax shield is equal to the risk-free rate r_f. All the tax shields in the subsequent years have to be discounted with the return to unlevered equity ρ. This is the Miles and Ezzell formulation, which we will discuss in more detail later.

M & M world

The unlevered value in year i is $V^{Un}(i)$, the levered value in year i is $V^L(i)$, the (levered) equity value in year i is $E^L(i)$, the value of debt in year i is $V^L(i)$ and the value of the tax shield in year i is $V^{TS}(i)$.

With perfect capital markets in an M & M world, we make the following assumptions. In any year i, the levered value is equal to the sum of the unlevered value and the value of the tax shield.

$$V^{L}(i) = V^{Un}(i) + V^{TS}(i) \qquad \text{(B4)}$$

Also, in any year i, the levered value is equal to the sum of the value of (levered) equity and value of debt.

$$V^{L}(i) = E^{L}(i) + D(i) \qquad \text{(B5)}$$

Combining line A4 and line A5, we obtain,

$$V^{Un}(i) + V^{TS}(i) = E^{L}(i) + D(i) \qquad \text{(B6)}$$

The expressions for the unlevered value, the (levered) equity value, the value of debt and the value of the tax shield are shown below. In any year i-1, the value is equal to the cash flow discounted by the appropriate discount rate.

$$V^{Un}(i-1) = \frac{FCF(i)}{1 + \rho_i} \qquad \text{(B7.1)}$$

$$E^{L}(i-1) = \frac{CFE(i)}{1 + e_i} \qquad \text{(B7.2)}$$

$$D(i-1) = \frac{CFD(i)}{1 + d_i} \qquad \text{(B7.3)}$$

$$V^{TS}(i-1) = \frac{TS(i)}{1 + \psi_i} \qquad \text{(B7.4)}$$

Substituting line A7.1 to line A7.4 in line A3, we obtain,

$$(1 + \rho_i)*V^{Un}(i-1) + (1 + \psi_i)*V^{TS}(i-1)$$

$$= (1 + e_i)*E^{L}(i-1) + (1 + d_i)*D(i-1) \qquad \text{(B8.1)}$$

Substituting line A6 into line A8.1 and simplifying, we obtain,

$$\rho_i*V^{Un}(i-1) + \psi_i*V^{TS}(i-1) = e_i*E^{L}(i-1) + d_i*D(i-1) \qquad \text{(B8.2)}$$

The weighted average cost of capital with the FCF

Let w_i be the WACC in year i based on the FCF(i). Then in year i-1, the levered value is equal to the FCF in year i discounted by w_i.

$$V^L(i-1) = \frac{FCF(i)}{1 + w_i} \tag{B9.1}$$

Rewriting line A9.1, we obtain that

$$FCF(i) = (1 + w_i)*V^L(i-1) \tag{B9.2}$$

From line A3, we know that

$$FCF(i) = CFE(i) + CFD(i) – TS(i) \tag{B10}$$

Substituting line A9.2, and line A7.2 to line A7.4 into line A10, we obtain,

$$(1 + w_i)*V^L(i-1) = (1 + e_i)*E^L(i-1) + (1 + d_i)*D(i-1) - (1 + \psi_i)*V^{TS}(i-1) \tag{B11}$$

Simplifying line A11.1 we obtain,

$$V^L(i-1) + w_i*V^L(i-1) = e_i*E^L(i-1) + d_i*D(i-1) - (1 + \psi_i)*V^{TS}(i-1) + E^L(i-1) + D(i-1) \tag{B12.1}$$

$$w_i*V^L(i-1) = e_i*E^L(i-1) + d_i*D(i-1) - (1 + \psi_i)*V^{TS}(i-1) \tag{B12.2}$$

We know that the tax shield in year i is equal to the tax rate τ times the cost of debt times the value of debt at the end of the previous year i-1.

$$TS(i) = \tau*d_i*D(i-1) \tag{B13}$$

Substituting line A7.4 and line A13 into line A12.2, we obtain the traditional formulation of the WACC.

$$w_i * V^L(i-1) = e_i * E^L(i-1) + d_i * D(i-1) - \tau * d_i * D(i-1)$$

(B14.1)

$$w_i = \frac{E^L(i-1)}{V^L(i-1)} * e_i + \frac{D(i-1)}{V^L(i-1)} * d_i * (1 - \tau) \qquad \text{(B14.2)}$$

The WACC is a weighted average of the cost of equity and the cost of debt, where the cost of debt is adjusted by the coefficient $(1 - \tau)$ and the weights are the market value of equity and market value of debt, as percentages of the levered market value. Line B14.2 is line 1 in the text.

Appendix C

List of symbols

ρ	The cost of the unlevered equity
d	The cost of debt (assumed constant)
D	Market value of debt
e_n	Levered cost of equity at year n
E^D	Market value of equity
ψ_n	Appropriate discount rate for tax savings at year n

Deriving e for a perpetuity

$$V^{AI} = \tau*d*D/\psi \qquad (C1a)$$

$$\psi*V^{AI} = \tau*d*D \qquad (C1b)$$

$$V^{SD} = FCF/\rho \qquad (C2a)$$

$$V^{SD}*\rho = FCF \qquad (C2b)$$

$$E^D = Z/e \qquad (C3a)$$

$$E^D *e = Z = FCF - d*D + \tau*d*D \qquad (C3b)$$

$$E^D *e = V^{SD}*\rho - d*D + \psi*V^{AI} \qquad (C4a)$$

$$E^D *e = [V^D - V^{AI}]\rho - d*D + \psi*V^{AI} \qquad (C4b)$$

$$e*E^D = \rho*E^D + (\rho - d)*D - (\rho - \psi)*V^{AI} \qquad (C4c)$$

$$e = \rho + (\rho - d)*D/E^D - (\rho - \psi)*V^{AI}/E^L \qquad (C4d)$$

Case 1

Assume $\psi = d$

$$e = \rho + (\rho - d)*D/E^D - (\rho - d)\ \tau D/E^D \qquad (C4e)$$

Reorganizing

$$e = \rho + (\rho - d)(1- \tau)D/E^D \qquad (C4f)$$

This line C4f is line (2) in the text. Deriving this formula for a finite horizon will produce a different equation for every period.

Case 2
Assume $\psi = \rho$
$e = \rho + (\rho - d)*D/E^D$ (C4g)

This is line (7) in the text. Deriving this formula for finite horizons we will get the same expression for every year.